2A: Introduction to Vectors

Arrows are used to illustrate vectors, arrows of same length and direction are the same vector (location doesn't matter)

vector can be written column $\begin{bmatrix} x \\ y \end{bmatrix}$ with mag $\sqrt{x^2+y^2}$

$\overrightarrow{AB}$ is a vector then $|\overrightarrow{AB}|$ is its magnitude.

Sample e.g. let $\underset{\sim}{u} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

find $\underset{\sim}{u} + \underset{\sim}{v}$ $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$v - u$ $\begin{bmatrix} 6 \\ -5 \end{bmatrix}$

$|v|$ $\sqrt{29}$

$-5v$ $\begin{bmatrix} -25 \\ 10 \end{bmatrix}$

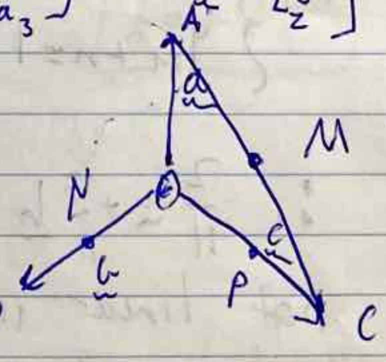
If $\underset{\sim}{v}$ is parallel to $\underset{\sim}{u}$, then $\underset{\sim}{v} = k\underset{\sim}{u}$

A position vector $\overrightarrow{OA}$ uses O as the origin and defines the point A.

In three dimensions use $\underset{\sim}{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ or $a = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

e.g. OABC OA = a OB = b OC = c

M N P midpoints



$AC = AO + OC = c - a$

$OM = OA + \frac{1}{2}AC = \frac{1}{2}a + \frac{1}{2}c$

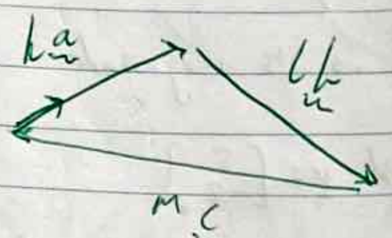
$CN = -\frac{1}{2}b + c$

$MN = MC + CN \quad \frac{1}{2}(c-a) + \frac{1}{2}b - c \quad \frac{1}{2}(b-a-c)$

$MP = \frac{1}{2}(c-a) - \frac{1}{2}c = \boxed{-\frac{1}{2}a}$

2A: Linear Independence

If a set of vectors are linearly dependant then one can be expressed as a linear combination of the others. This means there is a solution to $k\underset{\sim}{a} + l\underset{\sim}{b} + m\underset{\sim}{c} = 0$ other than $k=l=m=0$.



E.g. $\underset{\sim}{a} = \begin{bmatrix}1\\2\end{bmatrix}$, $\underset{\sim}{b} = \begin{bmatrix}-1\\3\end{bmatrix}$ and $\underset{\sim}{c} = \begin{bmatrix}1\\7\end{bmatrix}$ are linearly dependent since $c = 2a + b$ $\quad 2\underset{\sim}{a} + \underset{\sim}{b} - \underset{\sim}{c} = 0$ DEPENDANT

STEPS: First → Find m, n s.t.

$$m\underset{\sim}{a} + n\underset{\sim}{b} = \underset{\sim}{c}$$

e.g. Determine when linearly dependent

$a = \begin{bmatrix}3\\4\end{bmatrix}$ $\quad b = \begin{bmatrix}-2\\1\end{bmatrix}$ $\quad c = \begin{bmatrix}0\\1\end{bmatrix}$.

let $m\underset{\sim}{a} + n\underset{\sim}{b} = c$

$$\begin{cases} 3m - 2n = 0 \\ 4m + n = 1 \end{cases} \qquad 11m = 2 \qquad m = \frac{2}{11} \qquad n = \frac{3}{11}$$

$$\therefore \frac{2}{11}\underset{\sim}{a} + \frac{3}{11}\underset{\sim}{b} = \underset{\sim}{c}$$

not linear independant.

HAVE TO DEFINE FOR $ma + nb = c$

a, b, c non-parallel let the exist $m, n \in \mathbb{R} \setminus \{0\}$ such that $c = ma + nb$.

e.g. ~~Let~~ $a = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $c = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Let $ma + nb = c$

$$\begin{cases} 3m + 2n = -1 \\ 4m + n = 0 \\ -m + 3n = 1 \end{cases}$$

$5m = 1$

$m = \frac{1}{5}$, $n = -\frac{4}{5}$

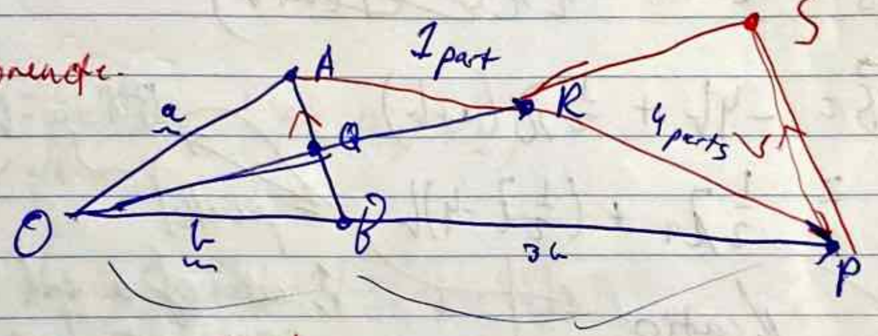
sub into (3)

$-\frac{1}{5} + 3(-\frac{4}{5}) = -\frac{13}{5} \neq 1$

$\therefore$ No values of m,n can be found $\therefore$ linearly independant.

If two work but doesn't sub into the third they are independent.

e.g. in onenote.



e.g. Find in terms of a b

$\vec{OQ}$

$\vec{AB} = b - a$ $\quad \vec{OQ} = OA + \frac{1}{2}AB$ $\quad a + \frac{1}{2}(b-a)$

$\boxed{\frac{1}{2}a + \frac{1}{2}b}$

$\vec{OR}$

$\vec{OR} = \frac{8}{5}OQ$ $\quad \vec{OR} = \frac{8}{5}(\frac{1}{2}a + \frac{1}{2}b)$

$\boxed{OR = \frac{4}{5}a + \frac{4}{5}b}$

$\vec{AR}$

$\vec{AR} = AO + OR$ $\quad -a + \frac{4}{5}a + \frac{4}{5}b = \boxed{-\frac{1}{5}a + \frac{4}{5}b}$

$\vec{RP}$

$\vec{RP} = -OR + OP$

$-\frac{4}{5}a + \frac{4}{5}b + 4b$

$\boxed{-\frac{4}{5}a + \frac{16}{5}b}$